Proof. It's necessary to prove that

$$(1+\sin 2x+\cos 2x)^2 = 4\cos^2 x\,(1+\sin 2x)\,. \tag{1}$$

Consider the left side of this equality. It's well known that $\sin 2x = 2\sin x\cos x$ and $\cos 2x = 2\cos^2 x - 1$. Having substituted these equalities to (1), we get the following:

$$(1+\sin 2x+\cos 2x)^2 = \left(1+2\sin x\cos x+2\cos^2 x-1\right)^2 = 4\cos^2 x\,(\sin x+\cos x)^2\,. \tag{2}$$

As $\sin^2 x+\cos^2 x = 1$, we have:

$$\begin{gathered} 4\cos^2 x\,(\sin x+\cos x)^2 = 4\cos^2 x\left(\sin^2 x+\cos^2 x+2\cos x\sin x\right) = \\ = 4\cos^2 x\,(1+2\cos x\sin x) = 4\cos^2 x\,(1+\sin 2x)\,. \end{gathered}$$

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