Answer on Question #43717 – Math - Algebra

If (m+1)th term of an AP is twice the (n+1)th term, prove that (3m+1)th term is twice the (m+n+1)th term.

Solution

By definition of arithmetical progression, (m+1)th and (n+1)th terms are

$a_{m+1} = a_1 + dm$, $a_{n+1} = a_1 + dn$ correspondingly.

By statement of the problem,

$a_{m+1} = 2a_{n+1}$, then

$a_1 + dm = 2(a_1 + dn)$, $a_1 = d(m - 2n)$.

Calculate (3m+1)th term:

$a_{3m+1} = a_1 + 3md = dm - 2dn + 3md = 4md - 2dn.$ **(1)**

Method 1

Calculate

$2a_{m+n+1} = 2(a_1 + md + nd) = 2a_1 + 2md + 2nd = 2md - 4nd + 2md + 2nd = 4md - 2nd$ **(2)**

Right-hand sides of (1) and (2) are the same, therefore left-hand sides of (1) and (2) are identical, that is, $a_{3m+1} = 2a_{m+n+1}$.

Method 2

We seek such q that

$a_{3m+1} = 2a_q$ **(3)**

Taking into account (1), rewrite (3) as follows

$$4md - 2dn = 2a_q$$

$$4md - 2dn = 2(a_1 + (q-1)d)$$

$$2md - dn = a_1 + (q-1)d$$

$$2md - dn = md - 2dn + (q-1)d$$

$$md + dn = (q-1)d$$

$$q = m + n + 1$$

We proved that (3) holds true when $q = m + n + 1$. It means that $a_{3m+1} = 2a_{m+n+1}$.