In a sample of 49 adolescents who served as the subjects in an immunologic study, one variable of interest was the diameter of a skin test reaction to an allergen. The sample mean and standard deviation were 21 and 11 mm erythema, respectively.

a. Use the Z or t-distribution? Why?

b. One-sided or two-sided test? Why?

c. Can it be concluded from these data that the population mean is less than 24 mm erythema?

**Solution.**

$x$ is diameter of skin test reaction to an antigen. Population is all the adolescents. $x \sim$ some distribution $(\mu, \sigma)$. Distribution is not known ($\mu \ and \ \sigma$ are not known).

$H_0: \mu = 30 \ mm \ ; H_1: \mu < 30 \ mm$ .

The alternative hypothesis contains "the population mean is less than", so it is One-sided test.

*Sample:*

$$n = 49 \ (a \ large \ sample), \bar{x} = 21, s = 11 \ mm.$$

*Test statistic*: Since distribution is not known and $n = 49 \geq 30$, sample size is large, we apply central limit theorem. Hence $\frac{\bar{x} - \mu}{s/\sqrt{n}}$ has an approximate standard normal distribution (Z-distribution).

$$z_{test} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{21 - 24}{\frac{11}{\sqrt{49}}} = -1.91.$$

Since the alternative hypothesis is left-tailed, the p-value is area to the left of -1.91.

$$\mathrm{p} - \mathrm{value} = 0.0281.$$

We can conclude from these data that the population mean is less than 24 mm erythema with Significance Level $\alpha > 0.0281.$

**Answer: a. Z-distribution; b. One-sided test; c. Yes, if Significance Level $\alpha > 0.0281.$**