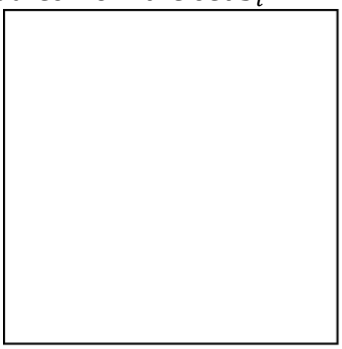
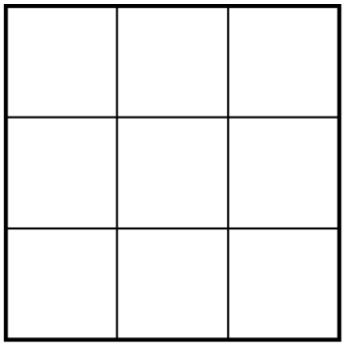
Answer on Question #43676, Math, Topology

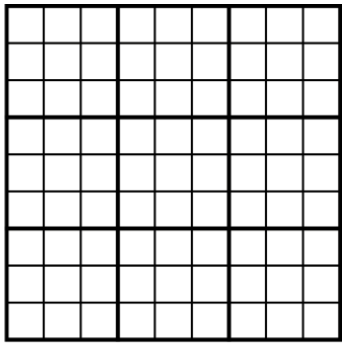
Problem. Find a Peano Curve which fills out the unit square in E^2
Solution. The Peano curve is the first example of a space-filling curve.
We present this example.
We will define the sequence of the sets of squares $_i$). Let S_0 is the single unit square. The set $_i$ is defined by partition all squares of $_{i-1}$ into 9 equals squares. Let P_i be the set of the centers of the squares from the set S_i.



S_0

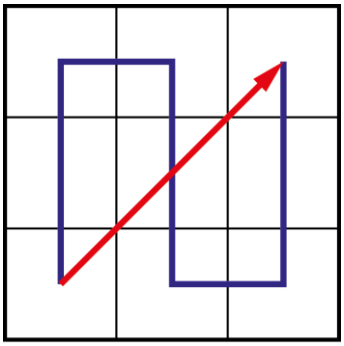


S_1



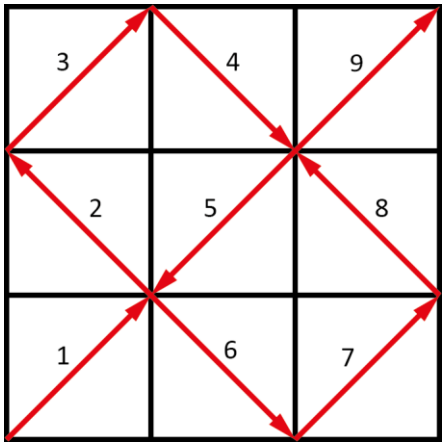
S_2

For each i we will define curve wich will connect the points from the set P_i with curve. The limit of such curve will be the Peano curve wich fills out the unit square. For each i we define the curve which connect left bottom corner and right top corner. For each i we could also obtain curve wich connect left top corner and right bottom corner by rotation. The curve for S_1 is shown in the following figure (blue line)



S_1

By red arrow we will denote the connection between corners.
Suppose that we have constructed curve for $i-1$, the curve for i we will construct by the following scheme



The order of the arrows is enumerated with the numbers.