Answer on Question #43673 - Math - Differential Calculus|Equation

Problem. Solve. (X^2-yz)p +(y^2-zx)q =z^2-xy

Remark.

We suppose that $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

We need to solve

$$(x^2 - yz)\frac{\partial z}{\partial x} + (y^2 - zx)\frac{\partial z}{\partial y} = z^2 - xy.$$

Solution.

The auxiliary equations are

$$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}.$$

Hence

$$\frac{dx - dy}{(x^2 - yz) - (y^2 - zx)} = \frac{dy - dz}{(y^2 - zx) - (z^2 - xy)} = \frac{dz - dx}{(z^2 - xy) - (x^2 - yz)}$$

or

$$\frac{d(x - y)}{(x - y)(x + y + z)} = \frac{d(y - z)}{(y - z)(x + y + z)} = \frac{d(z - x)}{(z - x)(x + y + z)}.$$

Therefore

$$\frac{d(x - y)}{x - y} = \frac{d(y - z)}{y - z} = \frac{d(z - x)}{z - x}$$

The solutions of the equations are

$$\ln|x - y| = \ln|y - z| + \ln C_1,$$
$$\ln|y - z| = \ln|z - x| + \ln C_2$$

or

$$\frac{x - y}{y - z} = C_1,$$
$$\frac{y - z}{z - x} = C_2.$$

So the general solution of the equation is

$$\phi\left(\frac{x - y}{y - z}, \frac{y - z}{z - x}\right) = 0,$$

where ϕ is a differentiable arbitrary function.

Answer: The general solution of equation is

$$\phi\left(\frac{x - y}{y - z}, \frac{y - z}{z - x}\right) = 0,$$

where ϕ is a differentiable arbitrary function.