

Answer on Question #43543-Math-Vector Calculus

Prove that

$$\left(\frac{\vec{a}}{a^2} - \frac{\vec{b}}{b^2}\right)^2 = \left(\frac{\vec{a} - \vec{b}}{ab}\right)^2.$$

Solution

1.

$$\begin{aligned}\left(\frac{\vec{a}}{a^2} - \frac{\vec{b}}{b^2}\right)^2 &= \left(\frac{\vec{a}}{a^2}\right)^2 + \left(\frac{\vec{b}}{b^2}\right)^2 - 2 \cdot \frac{\vec{a}}{a^2} \cdot \frac{\vec{b}}{b^2} = \frac{\vec{a}^2}{a^4} + \frac{\vec{b}^2}{b^4} - \frac{2(\vec{a}, \vec{b})}{a^2 b^2} = \frac{a^2}{a^4} + \frac{b^2}{b^4} - \frac{2(\vec{a}, \vec{b})}{a^2 b^2} \\ &= \frac{1}{a^2} + \frac{1}{b^2} - \frac{2(\vec{a}, \vec{b})}{a^2 b^2}.\end{aligned}$$

2.

$$\left(\frac{\vec{a} - \vec{b}}{ab}\right)^2 = \frac{(\vec{a} - \vec{b})^2}{a^2 b^2} = \frac{\vec{a}^2 + \vec{b}^2 - 2 \cdot \vec{a} \cdot \vec{b}}{a^2 b^2} = \frac{a^2 + b^2 - 2(\vec{a}, \vec{b})}{a^2 b^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{2(\vec{a}, \vec{b})}{a^2 b^2}.$$

That's why

$$\left(\frac{\vec{a}}{a^2} - \frac{\vec{b}}{b^2}\right)^2 = \left(\frac{\vec{a} - \vec{b}}{ab}\right)^2.$$