Question #43541, Math, Vector Calculus

Find the vector of magnitude $3\sqrt{2}$ which lies in zx plane and is at right angle to the vector $2\vec{\imath}+\vec{\jmath}+2\vec{k}$.

Solution.

We must find unknowing vector, wich lies in ZX plane.

Let $\vec{a}$ is the unknown vector and this vector lies in ZX plain. It can be written as:

$$\vec{a}=x\cdot\vec{\imath}+0\cdot\vec{\jmath}+z\cdot\vec{k}$$

where x and z some numbers. The y-coordinate of this vector is 0 because $\vec{a}$ lies in ZX plane.

Vector $\vec{a}$ is at right angle to the vector $2\vec{\imath}+\vec{\jmath}+2\vec{k}$, this means that

$$\vec{a}\cdot(2\vec{\imath}+\vec{\jmath}+2\vec{k})=0$$

We know that $|\vec{a}|=3\sqrt{2}$ then we can write the system of equations:

$$\begin{cases}|\vec{a}|=3\sqrt{2},\\ \vec{a}\cdot(2\vec{\imath}+\vec{\jmath}+2\vec{k})=0;\end{cases}\Rightarrow\begin{cases}\sqrt{x^2+z^2}=3\sqrt{2},\\ x\cdot2+0\cdot1+z\cdot2=0;\end{cases}\Rightarrow\begin{cases}x^2+z^2=18,\\ 2x+2z=0;\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}x^2+z^2=18,\\ x+z=0;\end{cases}\Rightarrow\begin{cases}x^2+z^2=18,\\ z=-x;\end{cases}\Rightarrow x^2+(-x)^2=18,\Rightarrow 2x^2=18,\Rightarrow$$

$$\Rightarrow x^2=9\Rightarrow\begin{cases}x=3,z=-3,\\ x=-3,z=3;\end{cases}$$

check out these answers:

a) $\begin{cases}\sqrt{3^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2},\\ (3\vec{\imath}-3\vec{k})\cdot(2\vec{\imath}+\vec{\jmath}+2\vec{k})=3\cdot2-3\cdot2=6-6=0\end{cases}$ ➔ it`s Ok

b) $\begin{cases}\sqrt{(-3)^2+3^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2},\\ (-3\vec{\imath}+3\vec{k})\cdot(2\vec{\imath}+\vec{\jmath}+2\vec{k})=-3\cdot2+3\cdot2=-6+6=0\end{cases}$ ➔ it`s Ok

Answer:

$$\vec{a}=\pm3\vec{\imath}\mp3\vec{k}=\begin{matrix}3\vec{\imath}-3\vec{k}\\ -3\vec{\imath}+3\vec{k}\end{matrix}$$