Answer on Question #43537, Math, Statistics and Probability

Let X be given by its "distribution" function $F(X)$ such that:

$$F(x) = \begin{cases} 0, if\ x \le 0 \\ \frac{x^2}{4}, if\ 0 < x \le 2 \\ 1, if\ x > 2 \end{cases}$$

Find $E(X)$, $var(X)$ and $std\ deviation\ (X)$.

Solution.

By definition,

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx$$

where $f(x) = F'(X)$. So,

$$f(X) = \begin{cases} 0, if\ x \notin (0,2] \\ \frac{x}{2}, if\ x \in (0,2] \end{cases}$$

Then,

$$E(X) = \int_{-\infty}^{0} x \cdot 0dx + \int_{0}^{2} \frac{x^2}{2} dx + \int_{2}^{\infty} x \cdot 0dx = 0 + \frac{2^3}{6} - 0 + 0 = \frac{4}{3}$$

By definition,

$$var(X) = \int_{-\infty}^{\infty} x^2 f(x)dx - \left(E(X)\right)^2 = \int_{-\infty}^{0} x^2 \cdot 0dx + \int_{0}^{2} \frac{x^3}{2} dx + \int_{2}^{\infty} x^2 \cdot 0dx - \frac{16}{9} =$$

$$= 0 + \frac{2^4}{8} + 0 - \frac{16}{9} = 2 - \frac{16}{9} = \frac{2}{9}$$

By definition,

$$std\ deviation(X) = \sqrt{var(X)}$$

So,

$$std\ deviation(X) = \frac{\sqrt{2}}{3}$$

Answer: $E(X) = 4/3$, $var(X) = 2/9$, $std\ deviation(X) = \sqrt{2}/3$.