Answer on Question #43494 – Math - Statistics and Probability

Find E(X) , var (X), and std deviation (x) if a random variable X is given by its density function f(x) , such that

f(x)=0, if x≤1

f(x)=(3/8) (x^2), if 0<x≤2

f(x)=0, if x>2

Remark.

We suppose the density function f(x) is defined by

$$f(x) = \begin{cases} 0, & x \le 0, \\ \dfrac{3}{8}x^2, & 0 < x \le 2, \\ 0, & x > 2, \end{cases}$$

as then $\int_{-\infty}^{+\infty} f(x)dx = 1$.

Solution.

The expected value equals

$$E(X) = \int_{-\infty}^{+\infty} xf(x)dx = \int_{0}^{2} xf(x)dx = \frac{3}{8}\int_{0}^{2} x^3 dx = \frac{3x^4}{32}|_0^2 = 1.5.$$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x)dx = \frac{3}{8}\int_{0}^{2} x^4 dx = \frac{3x^5}{40}|_0^2 = 2.4.$$

The variance equals

$$Var(X) = E(X^2) - E^2(X) = 2.4 - 2.25 = 0.15.$$

The standard deviation equals

$$\sigma(X) = \sqrt{Var(X)} \approx 0.387.$$

Answer: $E(X) = 1.5, Var(X) = 0.15, \sigma(X) = \sqrt{0.15} \approx 0.387.$