

Answer on Question #43478 – Math - Statistics and Probability

I have a biased coin that lands heads with probability p and start with an empty urn. I flip the coin n times. Each time the coin lands heads, I add a blue ball to the urn. Each time the coin lands tails, I add a green ball to the urn. After I finish flipping the coin and without knowing the composition of the urn, you draw k balls from the urn one at a time, replacing each ball you draw before drawing another one. If all k of the balls that you draw are blue, what is the probability that all n balls in the urn are blue?

Remark.

We suppose that we replace each ball with the ball of the same color.

Solution.

The probability that there are i blue ball in the urn equals $p(i) = \binom{n}{i} p^i (1-p)^{n-i}$ by Bernoulli trial. Suppose that there are i blue balls and $n - i$ green balls. The probability to select one blue ball is $\frac{i}{n}$.

We don't know the the composition of the urn, so the probability that the randomly selected ball from the urn is blue equals

$$\begin{aligned} p_1 &= \sum_{i=0}^n \frac{i}{n} p(i) = \sum_{i=0}^n \frac{i}{n} \binom{n}{i} p^i (1-p)^{n-i} = \\ &= \sum_{i=1}^n \binom{n-1}{i-1} p^i (1-p)^{n-i} = \\ &= \sum_{i=0}^{n-1} \binom{n-1}{i} p^{i+1} (1-p)^{n-1-i} = p. \end{aligned}$$

Hence the probability that we draw k blue balls is $p_k = p^k$.

This result could also be obtained from geometric distribution.

The probability that all n balls in the urn are blue equals $p_n = p^n$.

If all k of the balls that you draw are blue, then probability that all n balls in the urn are blue equals

$$P = \frac{p_n}{p_k} = p^{n-k}$$

from the formula of conditional probability.

Answer: $P = p^{n-k}$.