Let $f(x) = 2 - \frac{2}{x} + \frac{6}{x^2}$. Find the open intervals on which $f(x)$ is increasing (decreasing). Then determine the x -coordinates of all relative maxima (minima)

1. $f(x)$ is increasing on which intervals?
2. $f(x)$ is decreasing on which intervals?
3. the relative maxima of $f(x)$ occurs at x =?
4. the relative minima of $f(x)$ occurs at x =?

Solution:

First need the derivative of the function in order to find the critical points.

$$f'(x) = \frac{2}{x^2} - \frac{12}{x^3}$$

Simplifying this expression we obtain:

$$f'(x) = \frac{2x - 12}{x^3}$$

Therefore, the critical points will be those values of x which make the derivative equals zero or where it does not defined. So, we must solve the equation

$$\frac{2x - 12}{x^3} = 0$$

It's easy to identify the two critical points for this function (x_1 where the derivative equals 0, and $x_2$where the derivative doesn't exist).

$$x_1 = 6 \qquad\qquad x_2 = 0$$

For these two points, consider the following intervals

$$(-\infty, 0), \quad (0,6), \quad (6, \infty)$$

Choose the test points from each of the intervals and examine the sign of $f'(x)$

If $f'(x) > 0$, $f(x)$ is increasing in that interval.

If $f'(x) < 0$, $f(x)$ is decreasing in that interval.

$(-\infty, 0)$: test point chosen equal $x = -10$, $f'(-10) = \frac{-32}{-1000} = 0.032 > 0$, $f(x)$ increasing

$(0,6)$: test point chosen equal $x = 1$, $f'(1) = \frac{-10}{1} = -10 < 0$, $f(x)$ decreasing

$(6, \infty)$: test point chosen equal $x = 10$, $f'(10) = \frac{8}{1000} = 0.008 > 0$, $f(x)$ increasing

So $f(x)$ is increasing on$(-\infty, 0)$ and $(6, \infty)$

$f(x)$ is decreasing on $(0,6)$

Using the definitions of relative maxima (minima). If $f\,'(x) > 0$ ($f\,'(x) < 0$) on an open interval extending left from x_0 and $f\,'(x) < 0$ ($f\,'(x) > 0$) on an open interval extending right from x_0, then $f(x)$ has a relative maximum (minimum) at x_0.

$x_1 = 6$ relative minimum.

$x_2 = 0$ relative maximum.

Answer: **1)** $f(x)$ is increasing on $(-\infty, 0)$ and $(6, \infty)$

2) $f(x)$ is decreasing on $(0,6)$

3) the relative maxima of $f(x)$ occurs at x =0

4) the relative minima of $f(x)$ occurs at x =6