Answer on Question #43466 – Math – Calculus

Let $f(x) = 4x^3 + 5$. Find the open intervals on which f is increasing (decreasing). Then determine the – x coordinates of all relative maxima (minima).
1. f is increasing on the intervals?
2. f is decreasing on the intervals?
3. the relative maxima of f occurs at?
4. the relative minima of f occurs at?

Solution

Let's find the intervals of increase and decrease of the function $f(x)$. So, we need to find the values of x at which the derivative of the function is equal 0:

$$f'(x) = (4 * x^3 + 5)' = (4 * x^3)' + 5' = 4 * 3 * x^{3-1} + 0 = 12x^2$$

$f'(x) = 0;\ \ 12x^2 = 0$

$f'(x) = 0$; at $x = 0$;

$\begin{cases} f'(x) > 0,\ x \neq 0 \\ f'(x) = 0,\ x = 0 \end{cases}$ $\quad f(x) = \begin{cases} f'(x) > 0,\ x \in (-\infty; 0) \cup (0; +\infty) \\ f'(x) = 0,\ x = 0 \end{cases}$

Then, we can see, that:

1) function f(x) is increasing on the intervals: $x \in (-\infty; 0) \cup (0; +\infty)$

2) function f(x) is decreasing on the interval : $x \in \emptyset$.

3),4) this function have an extremum point at $x = 0$.

As $f'(x)$ has the same sign to the left and right of $x = x_0 = 0$ within the interval $(-3; 3)$, then $f(x_0)$ is neither a relative maximum nor minimum of f. In addition $f'(x_0) = 0$, then $(x_0, f(x_0))$ is called horizontal point of inflection of f.