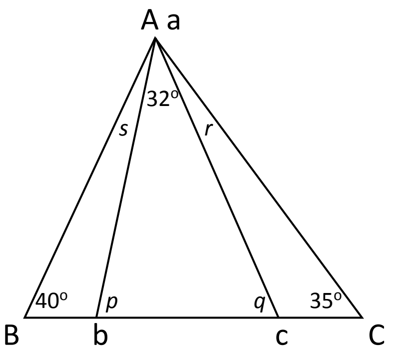
Answer on Question#43375 – Math – Geometry

Question

A small triangle abc lie in a large triangle ABC with same base and same top point i.e., a and A is same point. If <ABC=40 , <ACB=35 , <abc=p , <acb=q , <bac=32 , <BAb=s , <CAc=r, then find angle p, q, r, and s.

Solution



In ΔABC:

<BAC + <ABC + <ACB = 180°, whence

<BAC = 180° - <ABC - <ACB = 180° - 40° - 35° = 105°

<BAC = <BAb + <bAc + <cAC = *s* + 32° + *r* = 105°

s + *r* = 73°

In Δabc:

<bac + <acb + <cba = 180°, whence

<acb + <cba = *p* + *q* = 180° - <bac = 180° - 32° = 148°

In ΔBbA:

<ABb + <BbA + <BAb = 180°, hence <BbA = 180° - 40° - *s* = 140° - *s*

<BbA + <abc = 180°, i.e. (140° - *s*) + *p* = 180°, whence *p* - *s* = 40°

In ΔCcA:

<ACc + <CcA + <CAc = 180°, hence <CcA = 180° - 35° - *r* = 145° - *r*

<CcA + <acb = 180°, i.e. (145° - *r*) + *q* = 180°, whence *q* - *r* = 35°

In ΔAbC:

<ACb + <CbA + <bAC = 35° + *p* + (32° + *r*) = 180°, whence

p + *r* = 113°

In ΔABc:

<ABc + <BAc + <AcB = 40° + (*s* + 32°) + *q* = 180°, whence

q + *s* = 108°

Thus, we have the set of 6 linear equations with 4 unknown values:

$$\begin{cases} s + r = 73^\circ \\ p + q = 148^\circ \\ p - s = 40^\circ \\ q - r = 35^\circ \\ q + s = 108^\circ \\ p + r = 113^\circ \end{cases}$$

There is not the only solution of the set of equations.

Many solutions satisfying the condition may be found. For example

p = 53°, *q* = 95°, *r* = 60°, *s* = 13° or

p = 57°, *q* = 91°, *r* = 56°, *s* = 17° or

p = 90°, *q* = 58°, *r* = 23°, *s* = 50°

and so on.

Any of such solutions may be the answer.

Answer: *p* = 53°, *q* = 95°, *r* = 60°, *s* = 13°