

Answer on Question #43332 – Math - Statistics and Probability

X_1, X_2, \dots, X_{121} are independent and identically distributed random variables such that $E(X_i) = 3$ and $Var(X_i) = 25$. What is the standard deviation of their average? In other words, what is the standard deviation of $X = \frac{X_1 + X_2 + \dots + X_{121}}{121}$?

Solution

$$\begin{aligned} Var(X) &= Var\left(\frac{X_1 + X_2 + \dots + X_{121}}{121}\right) = \frac{1}{121^2} Var(X_1 + X_2 + \dots + X_{121}) \\ &= \frac{1}{121^2} (Var(X_1) + Var(X_2) + \dots + Var(X_{121})) = \frac{1}{121^2} (121 \cdot 25) = \frac{25}{121}. \end{aligned}$$

The standard deviation of their average is

$$\sigma(X) = \sqrt{Var(X)} = \sqrt{\frac{25}{121}} = \frac{5}{11}.$$

Answer: $\frac{5}{11}$.