Answer on Question #43289-Math-Multivariable Calculus

Differentiate this expression with respect to Z?

$$J = X \left\{ L + \left(\frac{LW}{Z}\right)^{\frac{p}{p-1}} \right\}^{-\left(\frac{1}{p}\right)}$$

Solution

$$\begin{split} \frac{\partial J}{\partial z} &= X \frac{\partial}{\partial z} \left\{ L + \left(\frac{LW}{Z}\right)^{\frac{p}{p-1}} \right\}^{-\left(\frac{1}{p}\right)} = X \left[-\left(\frac{1}{p}\right) \right] \left\{ L + \left(\frac{LW}{Z}\right)^{\frac{p}{p-1}} \right\}^{-\left(\frac{1}{p}\right)-1} \frac{\partial}{\partial z} \left\{ L + \left(\frac{LW}{Z}\right)^{\frac{p}{p-1}} \right\}. \\ \frac{\partial J}{\partial z} &= -\frac{\left(\frac{X}{p}\right)}{\left\{ L + \left(\frac{LW}{Z}\right)^{\frac{p}{p-1}} \right\}^{\frac{1+p}{p}}} \left(\frac{p}{p-1}\right) \left(\frac{LW}{Z}\right)^{\frac{p}{p-1}-1} \frac{\partial}{\partial z} \left(\frac{LW}{Z}\right) = -\frac{\left(\frac{X}{p-1}\right) \left(\frac{LW}{Z}\right)^{\frac{1}{p-1}}}{\left\{ L + \left(\frac{LW}{Z}\right)^{\frac{p}{p-1}} \right\}^{\frac{1+p}{p}}} \left(-\frac{LW}{Z^2} \right). \\ \frac{\partial J}{\partial z} &= \frac{\left(\frac{X}{(p-1)Z}\right) \left(\frac{LW}{Z}\right)^{\frac{1}{p-1}+1}}{\left\{ L + \left(\frac{LW}{Z}\right)^{\frac{p}{p-1}} \right\}^{\frac{1+p}{p}}} = \frac{\left(\frac{X}{(p-1)Z}\right) \left(\frac{LW}{Z}\right)^{\frac{p}{p-1}}}{\left\{ L + \left(\frac{LW}{Z}\right)^{\frac{p}{p-1}} \right\}^{\frac{1+p}{p}}}. \\ \frac{\partial J}{\partial z} &= \frac{\left(\frac{LW}{(p-1)Z}\right) \left(\frac{LW}{Z}\right)^{\frac{1+p}{p-1}}}{\left\{ L + \left(\frac{LW}{Z}\right)^{\frac{p}{p-1}} \right\}^{\frac{1+p}{p}}}. \end{split}$$

Answer:
$$\frac{\left(\frac{X}{(p-1)Z}\right)\left(\frac{LW}{Z}\right)^{\frac{p}{p-1}}}{\left\{L+\left(\frac{LW}{Z}\right)^{\frac{p}{p-1}}\right\}^{\frac{1+p}{p}}}.$$