Answer on Question #43248-Math-Statistics and Probability

We have a fair eight-sided die.

a. Find the math expectation of a single roll.

b. Find the math expectation of the numerical sum of 4 rolls.

c. Find the math expectation of the numerical product (i.e., multiplication) of 5 rolls.

Solution

Since the die is fair, the probability of any one of the eight values turning up on any single roll is $\frac{1}{8}$.

a. The math expectation of a single roll is then:

$$E(single\ roll) = 1\left(\frac{1}{8}\right) + 2\left(\frac{1}{8}\right) + 3\left(\frac{1}{8}\right) + 4\left(\frac{1}{8}\right) + 5\left(\frac{1}{8}\right) + 6\left(\frac{1}{8}\right) + 7\left(\frac{1}{8}\right) + 8\left(\frac{1}{8}\right) = \frac{36}{8} = \frac{9}{2}.$$

b. The math expectation of the numerical sum of 4 rolls is the sum of four math expectations of a single roll:

$$E(\text{ numerical sum of 4 rolls}) = \frac{9}{2} + \frac{9}{2} + \frac{9}{2} + \frac{9}{2} = 4 \cdot \frac{9}{2} = 18.$$

c. The math expectation of the numerical product (i.e., multiplication) of 5 rolls is the product of the math expectation of a single roll, multiplied by itself a total of five times:

$$E(\text{ numerical product of 5 rolls}) = \frac{9}{2} \cdot \frac{9}{2} \cdot \frac{9}{2} \cdot \frac{9}{2} \cdot \frac{9}{2} = \left(\frac{9}{2}\right)^5 = \frac{9^5}{2^5} \approx 1845.2812.$$