Answer on Question #43230 - Math - Other

Both P and NP are closed under operation a) union b) intersection

c) concatination

d) Kleene

Solution. Both sets are closed under all four operations. First we will show that *NP* is closed under all four operation.

Assume that $L_1, L_2 \in NP$. This means that there are nondeterministic deciders M_1 and M_2 such that M_1 decides L_1 in nondeterministic time $O(n^{k_1})$ and M_2 deciders L_2 in nondeterministic time $O(n^{k_2})$.

Intersection:

M = "On input w:

1. Run M_1 on w. If M_1 rejected then reject.

2. Else run M_2 on w. If M_2 rejected then reject.

3. Else accept."

The longest branch in any computation tree on input w of length n is $O(n^{\max(k_1,k_2)})$. So M is a poly-time nondeterministic decider for $L_1 \cap L_2$.

Union:

M = "On input w:

1. Run M_1 on w. If M_1 accepted then accept.

2. Else run M_2 on w. If M_2 accepted then accept.

3. Else reject."

The longest branch in any computation tree on input w of length n is $O(n^{\max(k_1,k_2)})$. So M is a poly-time nondeterministic decider for $L_1 \cup L_2$.

Concatenation:

M = "On input w:

1. Nondeterministically split w into w_1 , w_2 such that $w = w_1 w_2$.

2. Run M_1 on w_1 . If M_1 rejected then reject.

3. Else run M_2 on w_2 . If M_2 rejected then reject.

4. Else accept."

The longest branch in any computation tree on input w of length n is $O(n^{\max(k_1,k_2)})$. So M is a poly-time nondeterministic decider for $L_1 \circ L_2$.

Kleene star:

M = "On input w:

1. If w = e then accept.

2. Nondeterministically select a number m such that $1 \le m \le |w|$.

3. Nondeterministically split w into m pieces such that $w = w_1 w_2 \dots w_m$.

4. For all $i, 1 \le i \le m$: run M_1 on w_i . If M_1 rejected then reject.

5. Else (M_1 accepted all w_i , $1 \le i \le m$), accept."

The total running time is $O(n^{k_1+1})$. This means that M is a poly-time nondeterministic decider for L_1^* .

The same construction can be used to prove that P is closed under all four operation. **Answer:** Both sets are closed under all four operations.