Answer on Question # 43181- Math- Statistics and Probability

Professor Karl has n students in her classroom. At the end of the semester she decides to assign the students a group final project. If Professor Karl wants the students to pair into groups of 3, how many ways are there to choose the groups in which the order doesn't matter (the group Mike, John and Matt is the same as the group Matt, Mike and John).

Solution: We assume that if n doesn't divide by 3 then it is no way to assign the students into groups of 3. And the number of ways equals 0.

If n divides by 3 then the number of ways to choose the first group is $\binom{n}{3}$. The number of ways to choose the second group is $\binom{n-3}{3}$ and so on. So the number of ways to choose the groups is $\binom{n}{3} \cdot \binom{n-3}{3} \cdot \binom{n-6}{3} \cdot \ldots \cdot \binom{3}{3}$.

The number of such groups is n/3. As it doesn't matter what of these groups is the first or second, then we should divide our result by the number of permutations which is $\left(\frac{n}{3}\right)!$ So we obtain

$$\frac{1}{\left(\frac{n}{3}\right)!} \cdot \binom{n}{3} \cdot \binom{n-3}{3} \cdot \binom{n-6}{3} \cdot \ldots \cdot \binom{3}{3}.$$

Answer: $\begin{cases} \frac{1}{\left(\frac{n}{3}\right)!} \cdot \binom{n}{3} \cdot \binom{n-3}{3} \cdot \binom{n-6}{3} \cdot \ldots \cdot \binom{3}{3}, if\ n = 3k, k \in N \\ 0, if\ n \neq 3k, k \in N \end{cases}$, where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$