Answer on Question 43170, Math, Matrix | Tensor Analysis

$$X=\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad , \quad X^2=\begin{pmatrix} a^2+bc & ab+bd \\ ca+dc & cb+d^2 \end{pmatrix} \quad .$$

Thus, $X^2+2X=\begin{pmatrix} a^2+2a+bc & ab+2b+bd \\ ca+dc+2c & cb+d^2+2d \end{pmatrix}=-5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

This is a system of four non-linear equations for a,b,c,d .
Let us rewrite equations, which arise from diagonal elements in forms: $(a+1)^2+4+bc=0$ and $(d+1)^2+4+bc=0$. Subtracting one equation from another, obtain $(a+1)^2=(d+1)^2$, or $d=a$.
Plugging in $d=a$ into $ca+dc+2c=0$ gives $d=-1$. Hence, $a=d=-1$.
Substituting $a=-1$ into equation $a^2+2a+bc=-5$, obtain $bc=-4$, or $c=\frac{-4}{b}$.

Thus, $a=d=-1$, $c=\frac{-4}{b}$.

$$X=\begin{pmatrix} -1 & b \\ \frac{-4}{b} & -1 \end{pmatrix} \quad .$$