Let $n = 3m + k, k = \overline{0,2}, \forall n, m \in N$.

Then $n^2 = 9m^2 + 6mk + k^2 = \begin{Bmatrix} 9m^2 \\ 9m^2 + 6m + 1 \\ 9m^2 + 12m + 4 \end{Bmatrix} = \begin{Bmatrix} 3(3m^2) \\ 3(3m^2 + 3m) + 1 \\ 3(3m^2 + 4m + 1) + 1 \end{Bmatrix} \begin{matrix} k = 0 \\ k = 1 \\ k = 2 \end{matrix}$

Hence, $n^2 = 3l + k, k = \overline{0,1}, \forall n, l \in N$. But n is prime to 3, hence, n^2 is prime to 3 and in our case $n^2 = 3l + 1, \forall n, l \in N$.

Consider $x^2 + y^2 = (3u+1)^2 + (3v+1)^2 = 9u^2 + 6u + 1 + 9v^2 + 6v + 1 = 3(3u^2 + 3v^2 + 2u + 2v) + 2$.

Hence, $x^2 + y^2 = 3w + 2$. But we proved, that the square of natural number can have remainder equal to 0 or 1 and in our case it equal to 2. Then $x^2 + y^2$ can't be a perfect square.