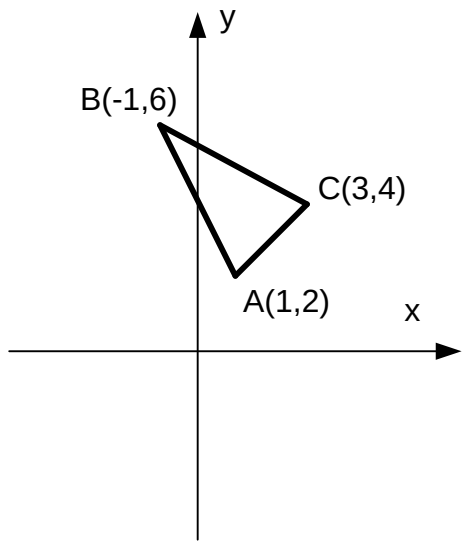
Answer on Question#42693 – Math – Calculus

Find the area of a triangle using the vertices (1,2),(-1,6),(3,4) use integration concept.

Solution:



The equation of line AB is:

$$\frac{x-1}{-1-1}=\frac{y-2}{6-2}$$

or $y_{AB}=-2x+4$

The equation of line BC is:

$$\frac{x+1}{3+1}=\frac{y-6}{4-6}$$

or $y_{BC}=-\frac{1}{2}x+\frac{11}{2}$

The equation of line CA is:

$$\frac{x-3}{1-3}=\frac{y-4}{2-4}$$

or $y_{CA}=x+1$

The area of a triangle is:

$$A=\int_{-1}^{1}(y_{BC}-y_{AB})\,dx+\int_{1}^{3}(y_{BC}-y_{CA})dx=$$

$$=\int_{-1}^{1}\left(-\frac{1}{2}x+\frac{11}{2}+2x-4\right)dx+\int_{1}^{3}\left(-\frac{1}{2}x+\frac{11}{2}-x-1\right)dx=$$

$$= \int_{-1}^{1} \left(\frac{3}{2}x + \frac{3}{2}\right) dx + \int_{1}^{3} \left(-\frac{3}{2}x + \frac{9}{2}\right) dx = \left(\frac{3}{4}x^2 + \frac{3}{2}x\right)\Big|_{-1}^{1} + \left(-\frac{3}{4}x^2 + \frac{9}{2}x\right)\Big|_{1}^{3} =$$

$$= \frac{3}{4} + \frac{3}{2} - \frac{3}{4} + \frac{3}{2} - \frac{27}{4} + \frac{27}{2} + \frac{3}{4} - \frac{9}{2} = 6$$