Answer on Question #42865 – Math - Linear Algebra

Find the inverse of the matrix

1 −1 0

2 −1 1

1 1 −1

using row reduction.

Solution

Form the augmented matrix $(A|E)$: $\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$.

$R_2 - 2R_1 \to R_2$: $\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$

$R_3 - R_1 \to R_3$: $\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 2 & -1 & -1 & 0 & 1 \end{array}\right)$

$R_3 - 2R_2 \to R_3$: $\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & -3 & 3 & -2 & 1 \end{array}\right)$

$\frac{R_3}{3} \to R_3$: $\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & -1 & 1 & -\frac{2}{3} & \frac{1}{3} \end{array}\right)$

$R_2 + R_3 \to R_2$: $\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & -1 & 1 & -\frac{2}{3} & \frac{1}{3} \end{array}\right)$

$R_2 + R_1 \to R_1$: $\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 & -1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & -1 & 1 & -\frac{2}{3} & \frac{1}{3} \end{array}\right)$

$-R_3 \to R_3$: $\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 & -1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & -1 & \frac{2}{3} & -\frac{1}{3} \end{array}\right)$

Answer: $\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} \\ -1 & \frac{1}{3} & \frac{1}{3} \\ -1 & \frac{2}{3} & -\frac{1}{3} \end{pmatrix}$.