Problem. Derive the matrix of translation in 2-dimensional plane. Why its first 2 column is identity matrix?

Solution. A translation is a function that moves every point a constant distance in a specified direction.

We rewrite vector $\begin{bmatrix} x \\ y \end{bmatrix}$ in 2-dimensinal using 3 coordinates as $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$. Then the translation of vector

$a = \begin{bmatrix} a_1 \\ a_2 \\ 1 \end{bmatrix}$ on vector $a = \begin{bmatrix} b_1 \\ b_2 \\ 1 \end{bmatrix}$ can be written using translation matrix $T_a = \begin{bmatrix} 1 & 0 & a_1 \\ 0 & 1 & a_2 \\ 0 & 0 & 1 \end{bmatrix}$ as

$$T_a b = \begin{bmatrix} 1 & 0 & a_1 \\ 0 & 1 & a_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ 1 \end{bmatrix} = a + b.$$

First column in translation matrix correspond to rotation and scaling of vectors. A translation just moves vectors, so first 2 columns in the matrix of translation is identity matrix.