Answer on Question #42819 – Math – Calculus

Question:

If, $\log(x^2+y^2) = \tan^{-1}\left(\frac{y}{x}\right)$. Find $\frac{dy}{dx}$.

Solution:

$$\log(x^2+y^2) = \tan^{-1}\left(\frac{y}{x}\right)$$

Differentiating both sides we get

$$\frac{d}{dx}\left(\log(x^2+y^2)\right) = \frac{d}{dx}\left(\tan^{-1}\left(\frac{y}{x}\right)\right)$$

Firstly let's consider the left term:

$$\frac{d}{dx}\left(\log(x^2+y^2)\right)$$

Using the chain rule, we obtain

$$\frac{d}{dx}\left(\log(x^2+y^2)\right) = \frac{2x+2yy'}{x^2+y^2}.$$

Now, let's consider the right term

$$\frac{d}{dx}\left(\tan^{-1}\left(\frac{y}{x}\right)\right)$$

Using the chain rule again and the quotient rule, we obtain

$$\frac{d}{dx}\left(\tan^{-1}\left(\frac{y}{x}\right)\right) = \frac{d}{dx}\left(\arctan\frac{y}{x}\right) = \frac{1}{\left(\frac{y}{x}\right)^2+1} * \left(\frac{xy'-y}{x^2}\right) = \frac{xy'-y}{y^2+x^2}$$

So,

$$\frac{2x+2yy'}{x^2+y^2} = \frac{xy'-y}{y^2+x^2}$$

Hence,

$$2x+2yy' = xy'-y$$

And

$$y'(x) = \frac{2\,x + y}{x - 2\,y}$$

Answer.

$$y'(x) = \frac{2\,x + y}{x - 2\,y}$$