Answer on Question #42809, Math, Linear Algebra

Problem.
Check whether the forms
2x^2+3y^2+5z^2−4xz−6yz

and 4x^2+3y^2+z^2−6xy−2xz

are orthogonally equivalent

Solution.
Two quadratic forms are called orthogonally equivalent, if there exists an orthogonal transformation from one to another. It is known that two quadratic forms are orthogonally equivalent if the characteristic polynomials of their matrixes are the same (since the orthogonal transformation doesn't change the characteristic polynomial of the matrix).

$$q_1 = 2x^2 + 3y^2 + 5z^2 - 4xz - 6yz$$
$$q_2 = 4x^2 + 3y^2 + z^2 - 6xy - 2xz$$

Their matrixes are A and B, respectively. $A = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 3 & -3 \\ -2 & -3 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -3 & -1 \\ -3 & 3 & 0 \\ -1 & 0 & 1 \end{pmatrix}$.

$$P_A(t) = \det(A - tE) = \begin{vmatrix} 2-t & 0 & -2 \\ 0 & 3-t & -3 \\ -2 & -3 & 5-t \end{vmatrix} = (2-t)(15 - 8t + t^2) - 2(6 - 2t)$$
$$= 30 - 16t + 2t^2 - 15t + 8t^2 - t^3 - 12 + 4t = -t^3 + 10t^2 - 27t + 18.$$
$$P_B(t) = \det(A - tE) = \begin{vmatrix} 4-t & -3 & -1 \\ -3 & 3-t & 0 \\ -1 & 0 & 1-t \end{vmatrix} = -1(3-t) + (1-t)(3 - 7t + t^2)$$
$$= -3 + t + 3 - 7t + t^2 - 3t + 7t^2 - t^3 = -t^3 + 8t^2 - 9t.$$

So, the equality $P_A(t) \equiv P_B(t)$ doesn't hold, hence these forms are not orthogonally equivalent and we are done.