Problem.

How many Boolean functions on two independent Boolean variables a and b are dependent on either a or b or both?

Solution.

There are $2^4 = 16$ different Boolean functions on two Boolean variables a and b (see table).

a	b	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Two of these functions are independent of both a and b, as they are constants:

$f_0(a, b) = 0;$

$f_{15}(a, b) = 1.$

Four of these are functions of a single variable:

$f_3(a, b) = a;$

$f_5(a, b) = b;$

$f_{10}(a, b) = \bar{b};$

$f_{12}(a, b) = \bar{a}.$

Ten other functions depend of both variables.

Answer: independent of both variables – 2 functions, dependent on either a or b – 4 functions, dependent of both variables – 10 functions.