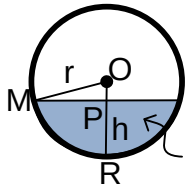
Answer on Question#42693 – Math – Geometry

Suppose a cylindrical tank is 12 feet long and 2 feet in diameter. The tank is laid on its side and filled with water to a height of 7 inches. Given that the volume of water in the tank is equal to the length of the tank times the cross-sectional area of the water, find the tank's volume.

Solution:



The volume of water in the tank is:

$V = Al$

Where A is the area of a segment and l is the length of tank

Given $r = 1\ foot$, since the radius is half the diameter, $h = 7/12\ feet$, $l = 12\ feet$

The area of a circular segment is:

$A = 2(A_{MOR} - A_{MOP})$

$A_{MOR} = \pi r^2 \frac{\angle MOR}{2\pi} = r^2 \frac{\angle MOR}{2}$

$\angle MOR = \cos^{-1}\frac{OP}{r} = \cos^{-1}\frac{r-h}{r} = \cos^{-1}\frac{1-\frac{7}{12}}{1} = \cos^{-1}\frac{5}{12}$

$A_{MOR} = 1^2 \frac{\cos^{-1}\frac{5}{12}}{2} = \frac{\cos^{-1}\frac{5}{12}}{2}$

$A_{MOP} = \frac{1}{2} OP \cdot PM$

$OP = r - h = 1 - \frac{7}{12} = \frac{5}{12}$

$PM = \sqrt{r^2 - (r-h)^2} = \sqrt{1 - \left(\frac{5}{12}\right)^2} = \frac{\sqrt{119}}{12}$

$A_{MOP} = \frac{1}{2} \cdot \frac{5}{12} \cdot \frac{\sqrt{119}}{12}$

$A = 2\left(\frac{\cos^{-1}\frac{5}{12}}{2} - \frac{1}{2} \cdot \frac{5}{12} \cdot \frac{\sqrt{119}}{12}\right) = \cos^{-1}\frac{5}{12} - \frac{5\sqrt{119}}{144} = 0.7622$

So the volume of water in the tank is:

$$\boldsymbol{V} = 0.7622 \cdot 12 = \mathbf{9.147}\ \boldsymbol{feet^3}$$