Answer on Question #42692 – Math - Trigonometry

If A, B, C, are the interior angles of a treangle ABC,show that cosec square A(B+C/2)- tan square A(A/2)=1.

Solution.

We need to prove that $\csc^2\frac{B+C}{2}-\tan^2\frac{A}{2}=1.$

First, rewrite $\csc\frac{B+C}{2}$ as $\frac{1}{\sin\frac{B+C}{2}}$.

Also notice that $\tan^2\frac{A}{2}=\frac{\sin^2\frac{A}{2}}{\cos^2\frac{A}{2}}=-1+\frac{1}{\cos^2\frac{A}{2}}$ (here we use that $\sin^2\alpha+\cos^2\alpha=1$).

After these steps we get $\csc^2\frac{B+C}{2}-\tan^2\frac{A}{2}=\frac{1}{\sin^2\frac{B+C}{2}}+1-\frac{1}{\cos^2\frac{A}{2}}$.

So, it suffices to prove that $\frac{1}{\sin^2\frac{B+C}{2}}=\frac{1}{\cos^2\frac{A}{2}}$.

Since $\sin\alpha=\sin(90°-\alpha)$ and $A+B+C=180°$, we get

$$\sin\frac{B+C}{2}=\sin\frac{180°-A}{2}=\sin\left(90°-\frac{A}{2}\right)=\cos\frac{A}{2}\Rightarrow\frac{1}{\sin^2\frac{B+C}{2}}=\frac{1}{\cos^2\frac{A}{2}}$$

$$\Rightarrow\frac{1}{\sin^2\frac{B+C}{2}}+1-\frac{1}{\cos^2\frac{A}{2}}=1.$$