Answer on Question #42600 – Math – Differential Calculus

Find the laplace tranform of the following 1-(t+5)^2cos10t, 2-te^-10t sin^2 t

Answer.

1. $f(t) = 1 - (t+5)^2 cos10t$

$$F(s) = L_t\{f(t)\}(s) = \int_0^\infty [1 - (t+5)^2 cos10t] e^{-st} dt =$$

$$\int_0^\infty [1 - t^2 cos10t - 10tcos10t - 25cos10t] e^{-st} dt,$$

and using well known Laplace transforms:

$L_t\{1\}(s) = \frac{1}{s}$. $L_t\{cos(at)\}(s) = \frac{s}{s^2+a^2}$,

$L_t\{t^n f(t)\}(s) = (-1)^n F^{(n)}(s)$, **(where** $F(s) = L_t\{f(t)\}(s)$**),**

so $L_t\{tcos(at)\}(s) = \frac{s^2-a^2}{(s^2+a^2)^2}$, $L_t\{t^2 cos(at)\}(s) = \frac{2s(s^2-3a^2)}{(s^2+a^2)^3}$,

finally, we have

$$F(s) = L_t\{f(t)\}(s) = \frac{1}{s} - \frac{25s}{s^2+100} - - \frac{10(s^2-100)}{(s^2+100)^2} - \frac{2s(s^2-300)}{(s^2+100)^3}.$$

2. $f(t) = 2 - te^{-10t} sin2t$

$$F(s) = L_t\{f(t)\}(s) = \int_0^\infty [\, 2 - te^{-10t} sin2t] e^{-st} dt$$

and using well known Laplace transforms:

$L_t\{1\}(s) = \frac{1}{s}$. $L_t\{e^{bt} sin(at)\}(s) = \frac{a}{(s-b)^2+a^2}$,

$L_t\{tf(t)\}(s) = -F'(s)$, **(where** $F(s) = L_t\{f(t)\}(s)$**),**

so $L_t\{te^{bt} sin(at)\}(s) = -\frac{2a(s-b)}{((s-b)^2+a^2)^2}$,

finally, we have $F(s) = L_t\{f(t)\}(s) = \frac{2}{s} - \frac{4(s+10)}{(s^2+20s+104)^2}$.