Answer on Question #42567, Math, Calculus

Problem.

Find the indicated limit, if it exists.

$$\lim_{x \to 0} f(x), \quad f(x) = \begin{cases} 5x - 8, & x < 0, \\ |-4 - x|, & x \geq 0. \end{cases}$$

a. -4

b. -12

c. -8

d. The limit does not exist

Solution.

Let us consider one-sided limits. If both of these limits are equal to a, then $\lim_{x \to 0} f(x) = a$.

Conversely, if they are not both equal to a, then the limit does not exist.

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0-} (5x - 8) = 5 \cdot 0 - 8 = -8.$$

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} |-4 - x| = |-4 - 0| = 4.$$

As $\lim_{x \to 0-} f(x) \neq \lim_{x \to 0+} f(x)$, then $\lim_{x \to 0} f(x)$ does not exist.

Answer: the limit does not exist.