Answer on Question #42551, Math, Topology

Problem. What is the basis of usual topology?

Solution. The usual topology T on $\mathbb{R}$ is defined by defining $U \subseteq \mathbb{R}$ to be in T if for every point $x \in U$, there is an $\varepsilon > 0$ such that $(x - \varepsilon, x + \varepsilon) \subseteq U$. This means that any open set in usual topology can be written as a union of the open intervals. Then all open intervals in the real line form a base for the usual topology.

Any rational (irrational) point can be approximated from the left or from the right with irrational (rational) points. This means that any open interval can be written as a union of the intervals with rational (irrational) endpoints. Then all open intervals in the real line with rational (irrational) endpoints form a base for the usual topology.

There also many other examples.

Answer.

- all open intervals;
- all open intervals with rational endpoints;
- all open intervals with irrational endpoints.