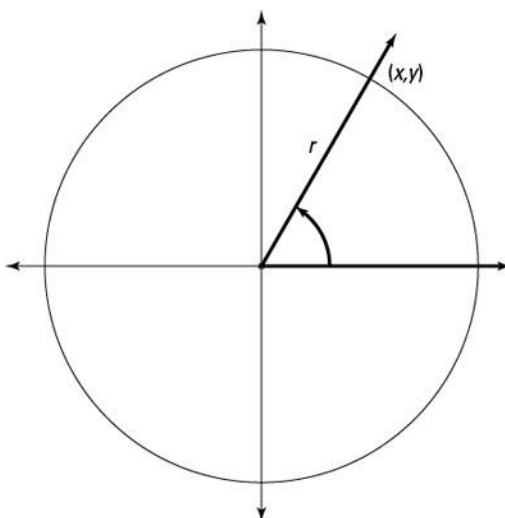**Solve.** Find the point on the terminal side of $\theta = -\dfrac{3\pi}{4}$ that has an x coordinate of -1.

Solution.

One way to find the values of the trig functions for angles is to use the coordinates of points on a circle that has its center at the origin. Letting the positive x-axis be the initial side of an angle, you can use the coordinates of the point where the terminal side intersects with the circle to determine the trigonometry functions.


The preceding figure shows a circle with a radius of r that has an angle drawn in standard position.

The equation of the circle is $x^2 + y^2 = r^2$. Based on this equation and the coordinates of the point where the terminal side of the angle intersects the circle, the six trig functions for angle θ are defined as follows:

$$\sin \theta = \frac{y}{r} \qquad \cos \theta = \frac{x}{r}$$

You can see where these definitions come from if you picture a right triangle formed by dropping a perpendicular segment from the point (x, y) to the x-axis.
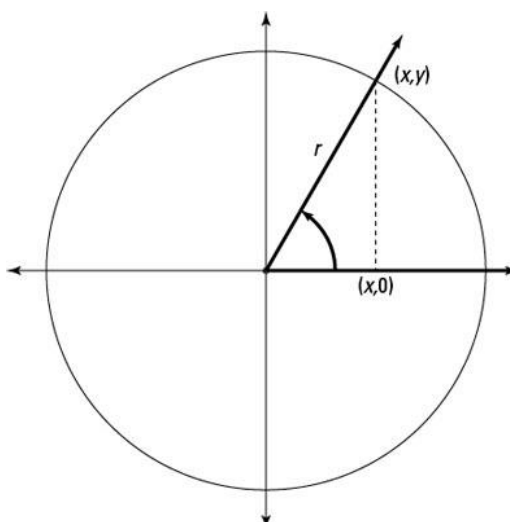
The preceding figure shows such a right triangle. Remember that the x-value is to the right (or left) of the origin, and the y-value is above (or below) the x-axis — and use those values as lengths of the triangle's sides. Therefore, the side opposite angle θ is y, the value of the y-coordinate. The adjacent side is x, the value of the x-coordinate. You find r, the radius of the circle, using the Pythagorean theorem.

$$r^2 = x^2 + y^2$$

$$r = 1 + y^2$$

$$\sin\frac{3\pi}{4} = \frac{y}{r}$$

$$\frac{1}{\sqrt{2}} = \frac{y}{r} = \frac{y}{1 + y^2}$$

$$y^2 - \sqrt{2}y + 1 = 0$$

$$y_1 = \frac{1 - i}{\sqrt{2}}$$

$$y_2 = \frac{1 + i}{\sqrt{2}}$$

Answer: $y_1 = \frac{1-i}{\sqrt{2}}$, $y_2 = \frac{1+i}{\sqrt{2}}$