Problem

Write a polynomial function of minimum degree with real coefficients whose zeros include those listed. Write the polynomial in standard form.

4, -14, and 5 + 8i.

Solution

The complex conjugate root theorem states that if P is a polynomial in one variable with real coefficients, and $a + bi$ is a root of P (a and b are real numbers), then its complex conjugate $a - bi$ is also a root of P.

That's why not only $5 + 8i$ is a zero of the desired polynomial, but also $5 - 8i$. Now, as we have 4 zeros of the polynomial desired (and it should also be of minimum degree possible), we can multiply 4 factors of the form $(x - x_0)$, where x_0 is a zero, together.

So, $P(x) = (x - 4)(x + 14)(x - 5 - 8i)(x - 5 + 8i) = (x^2 + 10x - 56)(x^2 - 10x + 89) =$
$= x^4 - 67x^2 + 1450x - 4984$.

Answer: $x^4 - 67x^2 + 1450x - 4984$.