Answer on Question #42499– Math – Abstract Algebra

Question:

Use mathematical induction to prove the statement is true for all positive integers n.

The integer n3 + 2n is divisible by 3 for every positive integer n.

Solution:

Basis: Show that the statement holds for n = 1.

$$n^3 + 2n = 1^3 + 2 * 1 = 3.$$

3 is divisible by 3, so the statement holds for n = 1.

Inductive step: Show that if the statement holds for n=k, then also the statement holds for n=k+1.

Assume that $k^3 + 2k$ is divisible by 3 (for some unspecified value of k). It must then be shown that $(k+1)^3 + 2(k+1)$ is divisible by 3 too, that is:

$$(k+1)^3 + 2(k+1) = k^3 + 3k^2 + 3k + 1 + 2k + 2 = k^3 + 2k + 3k^2 + 3k + 3$$
$$= (k^3 + 2k) + 3k^2 + 3k + 3 = (k^3 + 2k) + 3(k^2 + k + 1)$$

It can be easily seen that $(k^3 + 2k) + 3(k^2 + k + 1)$ is divisible by 3. It is because the first term $(k^3 + 2k)$ is divisible by 3 according to the inductive step. And the second term is divisible by 3, because it is a product of 3 and some integer number.