Answer on Question #42453 - Math - Statistics and Probability

Consider a random variable X such that P(x=1)=1/4 and P(x=0)=3/4. E(x)=1/4 and standard deviation is sqrt(3)/4. If X1, X2,..., Xn is a random sample of the population X, then lim n -> infinity of P(X1+X2+...+Xn/n > C) = 0 for every C>1/4.

Proof

Let $\bar{X}_n = \frac{X_1+X_2+\cdots+X_n}{n}$.

Compute $E(\bar{X}_n) = \frac{E(X_1)+E(X_2)+\cdots+E(X_n)}{n} = \frac{nE(X_1)}{n} = E(X_1) = \frac{1}{4}$,

$Var(\bar{X}_n) = Var\left(\frac{X_1+X_2+\cdots+X_n}{n}\right) = \frac{1}{n^2}Var(X_1+X_2+\cdots+X_n) = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n} = \frac{3}{16n}$,

Using Chebyshev`s inequality on $\bar{X}_n$ results in $P\{|\bar{X}_n - E(\bar{X}_n)| \geq \varepsilon\} \leq \frac{\sigma^2}{n\varepsilon^2}$.

$P\{|\bar{X}_n - E(\bar{X}_n)| \geq \varepsilon\} = P\{(\bar{X}_n \geq \varepsilon + E(\bar{X}_n)) \cup (\bar{X}_n \leq E(\bar{X}_n) - \varepsilon)\} \leq \frac{\sigma^2}{n\varepsilon^2}$.

$$0 \leq |\text{non} - \text{negativity of probability}| \leq P\{\bar{X}_n \geq \varepsilon + E(\bar{X}_n)\} \leq |monotonicity\ of\ probability| \leq$$

$$\leq P\{(\bar{X}_n \geq \varepsilon + E(\bar{X}_n)) \cup (\bar{X}_n \leq E(\bar{X}_n) - \varepsilon)\} \leq |\text{Chebyshev`s inequality on } \bar{X}_n| \leq \frac{\sigma^2}{n\varepsilon^2}$$

We obtain $P\{\bar{X}_n > C\} \leq \frac{\sigma^2}{n\varepsilon^2}$, for every $C > E(\bar{X}_n) = \frac{1}{4}$.

Pass to the limit and consider $0 \leq \lim_{n\to\infty} P\left(\frac{X_1+X_2+\cdots+X_n}{n} > C\right) \leq \lim_{n\to\infty} \frac{\sigma^2}{n\varepsilon^2} = 0, \varepsilon, C, \sigma^2$ are fixed.

By squeeze theorem we conclude

$\lim_{n\to\infty} P\left(\frac{X_1+X_2+\cdots+X_n}{n} > C\right) = 0$ for every $C > \frac{1}{4}$.

Q.E.D