Answer on Question #42433 – Math – Complex Analysis

Question. Express the complex $-3+3\sqrt{3}i$ number in trigonometric form.

Solution. A trigonometric form of a complex number $z = a+bi$ is a unique representation it in the following form:

$$z = r(\cos\phi + i\sin\phi),$$

where $r \geq 0$ and $0 \leq \phi < 2\pi$. From the identity

$$z = r(\cos\phi + i\sin\phi) = a + bi,$$

we see that

$$a = r\cos\phi, \qquad b = r\sin\phi,$$

and

$$a^2 + b^2 = r^2\cos^2\phi + r^2\sin^2\phi = r^2(\cos^2\phi + \sin^2\phi) = r^2,$$

and so

$$r = \sqrt{a^2+b^2}.$$

Let $-3+3\sqrt{3}i$. Then

$$r = \sqrt{(-3)^2 + (3\sqrt{3})^2} = \sqrt{9+27} = \sqrt{36} = 6.$$

Hence

$$z = -3+3\sqrt{3}i = 6\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right).$$

Notice also that

$$\cos\frac{2\pi}{3} = -\frac{1}{2}, \qquad \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2},$$

therefore

$$z = 6\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right).$$

Answer. $-3+3\sqrt{3}i = 6\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$.