Answer on Question #42424 – Math – Complex Analysis

Question. Express the complex $3 - 3i$ number in trigonometric form.

Solution. Recall that a trigonometric form of a complex number $z = a + bi$ is a unique representation it in the following form:

$$z = r(\cos\phi + i\sin\phi),$$

where $r \geq 0$ and $0 \leq \phi < 2\pi$. Since

$$z = r(\cos\phi + i\sin\phi) = a + bi,$$

we see that

$$a = r\cos\phi, \qquad b = r\sin\phi.$$

Therefore

$$a^2 + b^2 = r^2\cos^2\phi + r^2\sin^2\phi = r^2(\cos^2\phi + \sin^2\phi) = r^2 \cdot 1 = r^2,$$

and so

$$r = \sqrt{a^2 + b^2}.$$

For $z = 3 - 3i$ we have that

$$r = \sqrt{3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}.$$

Hence

$$z = 3 - 3i = 3\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right).$$

Since

$$\cos\frac{7\pi}{4} = \frac{1}{\sqrt{2}}, \qquad \sin\frac{7\pi}{4} = -\frac{1}{\sqrt{2}},$$

we obtain that

$$3 - 3i = 3\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right).$$

Answer. $z = 3\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$.