

Answer on Question #42403 – Math - Geometry

Task

State whether the given measurements determine zero, one, or two triangles.

$$A = 61^\circ, a = 23, b = 24$$

Solution

The law of cosines (also known as the cosine formula or cosine law)

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$23^2 = 24^2 + c^2 - 2 * 24 * c \cos 61^\circ$$

$$c^2 - 48 * 0.485 * c + 576 - 529 = 0$$

$$c^2 - 23.28c + 47 = 0$$

$$c = \frac{23.28 \pm \sqrt{23.28^2 - 4 * 47}}{2}$$

$$c = \frac{23.28 \pm 18.81}{2}$$

$$c = 21.045 \text{ or } c = 2.235$$

1) $c = 21.045$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$B = \sin^{-1} \frac{b \sin A}{a} = \sin^{-1} \frac{24 * 0.875}{23} = \sin^{-1} 0.91304 \dots = 65.93^\circ$$

$$C = 180^\circ - A - B = 180^\circ - 61^\circ - 65.93^\circ = 53.07^\circ$$

2) $c = 2.235$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\sin B = \sin(\pi - B) = \sin(180^\circ - B)$$

$$180^\circ - B = \sin^{-1} \frac{24 * 0.875}{23} = \sin^{-1} 0.91304 \dots = 65.93^\circ$$

$$B = 180^\circ - 65.93^\circ = 114.07^\circ$$

$$C = 180^\circ - A - B = 180^\circ - 61^\circ - 114.07^\circ = 4.93^\circ$$

Answer:

1) $A = 61^\circ$	$a = 23,$
$B = 65.93^\circ$	$b = 24$
$C = 53.07^\circ$	$c = 21.045$

2) $A = 61^\circ$	$a = 23,$
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$$B = 114.07^\circ$$

$$C = 4.93^\circ$$

$$b = 24$$

$$c = 2.235$$

There are 2 triangles.