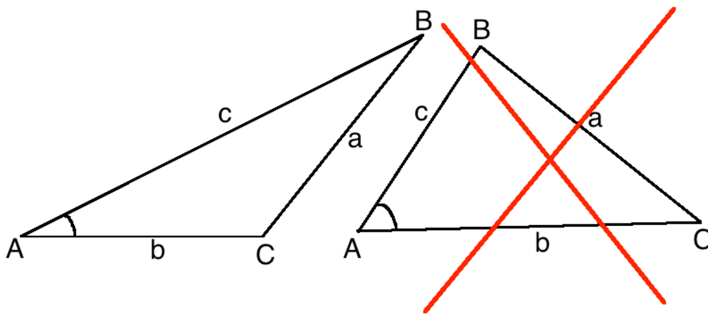
Answer on Question #42400 – Math – Geometry

Solve the triangle.

A = 33°, a = 19, b = 14

Can this solved how ?

Solution:



$\mathrm{A} = 33°, \mathrm{a} = 19, \mathrm{b} = 14$

Solving the triangle means to finding missing sides and angles (side c, angles C and B)
Law of Sines (the Sine Rule):

$$\frac{\mathrm{a}}{\sin \mathrm{A}} = \frac{\mathrm{b}}{\sin \mathrm{B}}$$

$$\sin \mathrm{B} = \frac{\mathrm{b}}{\mathrm{a}} \sin \mathrm{A} = \frac{14}{19} \cdot \sin 33° \Longrightarrow \mathrm{B} = \arcsin\left(\frac{\mathrm{b}}{\mathrm{a}} \sin \mathrm{A}\right) =$$

$$= \arcsin\left(\frac{14}{19} \cdot \sin 33°\right) = 24°$$

$$\text{or } B = 180° - \arcsin\left(\frac{14}{19} \cdot \sin 33°\right) = 156°$$

Thus, we must consider two cases : $B = 24°\ and\ B = 156°$

#1 $(B = 24°)$

The angles always add to 180°: when you know two angles you can find the third:

$$\mathrm{A} + \mathrm{B} + \mathrm{C} = 180°$$

$$\mathrm{C} = 180° - \mathrm{A} - \mathrm{B} = 180° - 33° - 24° = 123°$$

To find side c we can use law of sines again, but with side c and angle C:

$$\frac{\mathrm{a}}{\sin \mathrm{A}} = \frac{\mathrm{c}}{\sin \mathrm{C}}$$

$$\mathrm{c} = \frac{\sin \mathrm{C}}{\sin \mathrm{A}} \mathrm{a} = 19 \cdot \frac{\sin(123°)}{\sin(33°)} = 29$$

#2 $(B = 156°)$

The angles always add to 180°: when you know two angles you can find the third:

$$\mathrm{A} + \mathrm{B} + \mathrm{C} = 180°$$

$$\mathrm{C} = 180° - \mathrm{A} - \mathrm{B} = 180° - 33° - 156° = -9°$$

Angle can not be negative, so the second case is not possible.

Answer: $\mathrm{B} = 24°, \mathrm{C} = 123°, \; \mathrm{c} = 29.$