Answer on Question #42396 – Math – Trigonometry

Find all solutions in the interval $[0,2\pi)$:

$$7\tan 3x - 21\tan x = 0.$$

Solution.

$$\tan 3x = \frac{\sin 3x}{\cos 3x} = \frac{3\sin x - 4\sin^3 x}{4\cos^3 x - 3\cos x} = \tan x \cdot \frac{3 - 4\sin^2 x}{4\cos^2 x - 3} = \tan x \cdot \frac{4\cos^2 x - 1}{4\cos^2 x - 3} =$$

$$= \tan x \cdot \frac{\frac{4}{\tan^2 x + 1} - 1}{\frac{4}{\tan^2 x + 1} - 3} = \tan x \cdot \frac{3 - \tan^2 x}{1 - 3\tan^2 x};$$

It follows $tg^2x + 1 = \frac{1}{cos^2x}$ from $sin^2x + cos^2x = 1$, therefore $cos^2x = \frac{1}{tg^2x+1}$.

Solve

$$7\tan 3x - 21\tan x = 0 \Rightarrow \tan x \cdot \frac{3 - \tan^2 x}{1 - 3\tan^2 x} - 3\tan x = 0 \Rightarrow \left[\begin{array}{c} \tan x = 0 \\ \frac{3 - \tan^2 x}{1 - 3\tan^2 x} - 3 = 0 \end{array}\right. \Rightarrow$$

$$\Rightarrow \left[\begin{array}{c} \tan x = 0 \\ 3 - \tan^2 x = 3 - 9\tan^2 x \end{array}\right. \Rightarrow \tan x = 0;$$

$$\begin{cases} \tan x = 0 \\ x \in [0,2\pi) \end{cases} \Rightarrow \left[\begin{array}{l} x = 0 \\ x = \pi \end{array}\right..$$

Answer. $\{0; \pi\}$.