Answer on Question#42374 - Math - Multivariable Calculus

Show that the function $F(x,y) = (e^{xy},\ \ln x)$ is locally invertible at (1,3) .

Solution:

The function $F(x,y)$ is locally invertible at (1,3) if exists $F^{-1}(x,y)$ and uniquely defined $F(x,y)$ at (1,3).

Let us find $F^{-1}(x,y)$:

if $F(x,y) = (a,b),$ so $F^{-1}(a,b) = (x,y).$

in our case, $a = e^{xy}$, $b = \ln x$;

so,

$\boldsymbol{x = e^b}$;

$xy = \ln a;$

$\boldsymbol{y = \ln a / e^b}$

The inverse function is:

$F^{-1}(a,b) = (x,y) = (e^b, \ln a/e^b)$

So, $F(1,3) = (e^{1*3}, \ln 1) = (e^3, 0)$

$F^{-1}(1,3) = (e^3, ln1/e^3) = (e^3, 0)$

$F(1,3)$ and $F^{-1}(1,3)$ exist and are uniquely defined , so the function $F(x,y) = (e^{xy},\ \ln x)$ is locally invertible at (1,3).