Answer on Question #42372 – Math - Integral Calculus

The density at any point on a semicircular lamina is proportional to the distance from the centre of the circle. Find the centre of gravity of the lamina.

Solution.

Let lamina occupies the region D. Suppose that its density at a point (x, y) in D is $\rho(x, y)$. Then the total mass is

$$m = \iint_D \rho(x, y)dA$$

Let $(\bar{x}, \bar{y})$ be the center of mass, then

$$\bar{x} = \frac{1}{m}\iint_D x\rho(x, y)dA, \quad \bar{y} = \frac{1}{m}\iint_D y\rho(x, y)dA,$$

where $\iint_D y\rho(x, y)dA$ and $\iint_D x\rho(x, y)dA$ are the moments about x- and y- axises.

Use polar coordinates to find $(\bar{x}, \bar{y})$. We have $\rho \propto r$ or $\rho = kr$.

Find m:

$$m = \int_0^\pi \int_0^1 kr \cdot rdrd\theta = \pi \cdot \frac{1}{3} \cdot k = \frac{k\pi}{3}$$

Find M_x and M_y:

$$M_x = \int_0^\pi \int_0^1 (r \sin\theta)(kr)rdrd\theta = 2 \cdot k \cdot \frac{1}{4} = \frac{k}{2}$$

$$M_y = \int_0^\pi \int_0^1 (r \cos\theta)(kr)rdrd\theta = 0 \cdot \int_0^1 kr^3 dr = 0$$

Thus

$$(\bar{x}, \bar{y}) = \left(0, \frac{3}{2\pi}\right)$$

Answer:

$$(\bar{x}, \bar{y}) = \left(0, \frac{3}{2\pi}\right)$$