Answer on Question #42336 – Math – Calculus

Determine the most economical dimensions of an open-air swimming pool of volume 32 metre cube with a square bottom so the facing of its wall and bottom require the least quantity of material.

Solution.

Let the length of the side of the bottom be equal to a. Then, the height of the pool is $\frac{32}{a^2}$.

The square of four walls and the bottom is $S(a) = 4 \cdot \left(\frac{32}{a^2} \cdot a\right) + a^2 = \frac{128}{a} + a^2$, $a > 0$.

Let`s minimize $S(a)$.

The derivative of $S(a)$.: $S'(a) = -\frac{128}{a^2} + 2a = \frac{2(a^3 - 64)}{a^2}$.

Find roots of the equation $S'(a) = 0$: $\frac{2(a^3 - 64)}{a^2} = 0$, $a^3 = 64$, $a = 4$.

As the derivative $S'(a)$ changes the sign from minus to plus as a passes through the point $a = 4$, the function $S(a)$ has a minimum at this point.

So, the most economical dimensions of the pool are the following: the bottom with a side, which equals to 4 meters, and the wall of height, which equals to $\frac{32}{4^2} = 2$ meters.

Answer: $4 \times 4 \times 2$ meters.