Question. Let U and V be open dense subsets of X. Prove that U intersection V is also dense in X.

Proof. Recall that a subset $U \subset X$ is *dense* if for every non-empty open $W \subset X$ the intersection $U \cap W \neq \varnothing$.

Now let $W \subset X$ be any non-empty open subset. We should prove that $W \cap (U \cap V) \neq \varnothing$. Since U is dense in X, we have that

$$W \cap U \neq \varnothing.$$

But $W \cap U$ is open as an intersection of two open subsets. Therefore

$$(W \cap U) \cap V = W \cap (U \cap V) \neq \varnothing,$$

since V is dense as well.