Answer on Question #42293-Math-Trigonometry

Prove that

$$\frac{\text{sinB}}{\text{sinA}} = \frac{\sin(2\text{A} + \text{B})}{\text{sinA}} - 2\cos(\text{A} + \text{B}).$$

Solution

$$\sin(2\text{A} + \text{B}) = \sin(\text{A} + (\text{A} + \text{B})) = \sin(\text{A})\cos(\text{A} + \text{B}) + \sin(\text{A} + \text{B})\cos(\text{A}).$$

$$\frac{\sin(2\text{A} + \text{B})}{\text{sinA}} - 2\cos(\text{A} + \text{B}) = \frac{\sin(\text{A})\cos(\text{A} + \text{B}) + \sin(\text{A} + \text{B})\cos(\text{A})}{\text{sinA}} - 2\cos(\text{A} + \text{B})$$

$$= \frac{\sin(\text{A})\cos(\text{A} + \text{B})}{\text{sinA}} + \frac{\sin(\text{A} + \text{B})\cos(\text{A})}{\text{sinA}} - 2\cos(\text{A} + \text{B})$$

$$= \cos(\text{A} + \text{B}) + \frac{\sin(\text{A} + \text{B})\cos(\text{A})}{\text{sinA}} - 2\cos(\text{A} + \text{B}) = \frac{\sin(\text{A} + \text{B})\cos(\text{A})}{\text{sinA}} - \cos(\text{A} + \text{B})$$

$$= \frac{\sin(\text{A} + \text{B})\cos(\text{A}) - \text{sinA}\cos(\text{A} + \text{B})}{\text{sinA}} = \frac{\sin\big((\text{A} + \text{B}) - A\big)}{\text{sinA}} = \frac{\text{sinB}}{\text{sinA}}.$$