**Answer on Question #42266 – Math - Statistics and Probability**

It is not known whether a coin is fair or unfair. If the coin is fair, the probability of a tail is 0.5, but if the coin is unfair, the probability of a tail is 0.1. If the probability of a fair coin is 0.8 and the probability of an unfair coin is 0.2, the coin is tossed once, and a tail is the result:

1. What is the probability that the coin is fair?

2. What is the probability that the coin is unfair?

**Solution**

Let the event 'fair coin' by designated $A_1$ and the event 'unfair coin' by $A_2$. Then the given information be put as under:

$$\begin{cases} P(A_1) = 0.8 \\ P(A_2) = 0.2 \end{cases} \textit{A prior (or unconditional) probabilities.}$$

$$\begin{cases} P(tail|A_1) = 0.5 \\ P(tail|A_2) = 0.1 \end{cases} \textit{Conditional probabilities.}$$

$$\begin{cases} P(tail\ and\ A_1) = P(A_1) \cdot P(tail|A_1) = 0.8 \cdot 0.5 = 0.4 \\ P(tail\ and\ A_2) = P(A_2) \cdot P(tail|A_2) = 0.2 \cdot 0.1 = 0.02 \end{cases} \textit{Joint probabilities.}$$

A tail can occur in combination with 'fair coin' or in combination with 'unfair coin'. The probability of the former is 0.4 and of the latter it is 0.02. The sum of the probabilities would result in the unconditional probability of a tail on the first toss i.e.,

$$P(tail) = 0.4 + 0.02 = 0.42.$$

Thus if a tail occurs and if it is not known whether the coin tossed once is fair or unfair coin, then the probability of its being a fair coin is:

$$P(A_1|tail) = \frac{P(tail\ and\ A_1)}{P(tail)} = \frac{0.4}{0.42} = 0.95.$$

This is the posterior (or revised) probability of a fair coin (or $A_1$) given that tail is the result in the first toss of a coin obtained through Bayes's Rule.

We can similarly calculate the posterior probability of a unfair coin (or $A_2$) given that tail is the result in the first toss and it can be shown as follows:

$$P(A_2|tail) = \frac{P(tail\ and\ A_2)}{P(tail)} = \frac{0.02}{0.42} = 0.05.$$

Thus the revised probabilities after one toss when the toss results in tail are 0.95 of a fair coin and 0.05 of an unfair coin.

**Answer: 1. 0.95; 2. 0.05.**