# Answer on Question #42209, Math, Topology

**Problem.** Prove that intersection of two dense subsets is again dense subset.
**Counterexample.** The sets $\mathbb{Q}$ and $\mathbb{Q}+\sqrt{3}$ are dense in $\mathbb{R}$ with the usual topology, but their intersection $\mathbb{Q} \cap (\mathbb{Q}+\sqrt{3}) \neq \emptyset$ isn't dense in $\mathbb{R}$.
**Solution.** This fact is true if only one of this subsets is open. Let $(X, T)$ be a topological space. Suppose that $U$ is a dense open subset of $X$ and $D$ is any dense subset of $X$. If $V \in T$ is a non-empty open set in $X$, then $V \cap U \in T$ is a non-empty open set. $D$ is a dense subset of $X$, so $(V \cap U) \cap D \neq \emptyset$. Hence, $V \cap (U \cap D) \neq \emptyset$ and indeed $U \cap D$ is dense in $X$.