

Answer on Question #42158 – Math -Real Analysis

Is the function $f: \mathbb{R}^2 \setminus \{0,0\} \rightarrow \mathbb{R}^2$ defined by $f(x, y) = \left(\frac{x}{x^2+y^2}, -\frac{y}{x^2+y^2} \right)$ continuous on $\mathbb{R}^2 \setminus \{0,0\}$?

Solution.

Theorem. Let $f: A \subset \mathbb{R}^n \rightarrow \mathbb{R}^m$ be given by

$$f(x) = (f_1(x); \dots; f_m(x)).$$

Then f is continuous at a $a \in A$ if and only if f_i is continuous at a for $i = 1, 2, \dots, m$.

So it is enough to show that functions $f_1(x, y) = \frac{x}{x^2+y^2}$ and $f_2(x, y) = -\frac{y}{x^2+y^2}$ are continuous on $\mathbb{R}^2 \setminus \{0,0\}$. They are continuous as fraction of the two continuous functions.

Answer: Yes, It is.