

Answer for Question #42115, Math, Statistics and Probability

Problem:

A manager claimed that his employees perform better at work with music playing. An experiment was conducted to verify the manager's claim. One day, the music was turned on during working hours with 45 employees. It is found that the mean productivity level is 5.2 with standard deviation of 2.4. On a different day, the music was turned off and there were 40 employees with a mean productivity level of 4.8 and a standard deviation 1.2. Does the experiment support the manager's claim? Use 5% level of significance to solve.

$$n_1 = 45; \mu_1 = 5.2; \sigma_1 = 2.4;$$

$$n_2 = 40; \mu_2 = 4.8; \sigma_2 = 1.2;$$

Solution:

The solution to this problem takes four steps: (1) state the hypotheses, (2) formulate an analysis plan, (3) analyze sample data, and (4) interpret results.

State the hypotheses. The first step is to state the null hypothesis and an alternative hypothesis.

Null hypothesis:

$$\mu_1 - \mu_2 > 0$$

Alternative hypothesis:

$$\mu_1 - \mu_2 \leq 0$$

Note that these hypotheses constitute a one-tailed test. The null hypothesis will be rejected if the mean difference between sample means is too small.

- **Formulate an analysis plan.** For this analysis, the significance level is 0.05. Using sample data, we will conduct a [two-sample t-test](#) of the null hypothesis.
- **Analyze sample data.** Using sample data, we compute the standard error (SE), and the t-score test statistic (t).

$$SE = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = 0.405$$

$$t = \frac{\mu_1 - \mu_2}{SE} = 0.988$$

For this **one-tailed test**, the **P-value** is the probability of obtaining a t-score test statistic that is more extreme than 0.988 (i.e., greater than 0.988), assuming the null hypothesis is true. If the P-value is less than the significance level, we reject the null hypothesis.

To find the probability that the t-score test statistic is greater than 0.988, we use the **t Distribution Calculator** or a table (with degrees of freedom = 39). The calculator tells us that $P(T \leq 0.988) = 0.84$. Thus, the P-value is:

$$P(T \geq 0.988) = 1 - P(T \leq 0.988) = 0.16$$

- **Interpret results.** Since the P-value (**0.16**) is greater than the significance level (0.05), we cannot reject the null hypothesis. It means that the claim is supported.

Answer: the claim is supported.