Answer on Question #42088, Math, Statistics and Probability

An unbiased coin is tossed twice. The four possible outcomes are equiprobable. If A is the event "both head or tail have occurred" and B is the event: "at most one tail is observed", then find P(A), P(B), P(A/B) and P(B/A)?

Solution:

Experiment - a set of conditions:

- 1) Coin is unbiased
- 2) Coin is tossed twice

Denote events: head = 1

tail = 0

Elementary events: $\omega = (a, b)$, where $a, b \in \{0,1\}$ – permutation with repetitions

Then space of elementary events: $\Omega = \{\omega = (a, b)\} \Rightarrow |\Omega| = 2^2 = 4 < \infty$

Take the maximal $\sigma - algebra \ \mathfrak{A} = \mathfrak{A}_{max}$

Then probability $\mathbb{P}(\cdot)$ – classical type.

$$A = \{(0,1), (1,0)\} \Rightarrow |A| = 2 \Rightarrow \mathbb{P}(A) = \frac{|A|}{|\Omega|} = \frac{2}{4} = \frac{1}{2}$$
$$B = \{(0,1), (1,0), (0,0)\} \Rightarrow |B| = 3 \Rightarrow \mathbb{P}(B) = \frac{|B|}{|\Omega|} = \frac{3}{4}$$
$$A \cap B = \{(0,1), (1,0)\} \Rightarrow |A \cap B| = 2 \Rightarrow \mathbb{P}(A \cap B) = \frac{|A \cap B|}{|\Omega|} = \frac{2}{4} = \frac{1}{2}$$
$$\mathbb{P}(A/B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{4}{6} = \frac{2}{3}$$
$$\mathbb{P}(B/A) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(A)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

Answer:

1)
$$\mathbb{P}(A) = \frac{1}{2}$$

2) $\mathbb{P}(B) = \frac{3}{4}$
3) $\mathbb{P}(A/B) = \frac{2}{3}$
4) $\mathbb{P}(B/A) = 1$